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Engineering Notes

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Generalized Empirical Airfoil Stagnation Point Location Prediction

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Introduction

A GENERALIZED method for the prediction of a stagnation point location as a function of the angle of attack on symmetrical and cambered wing profiles as well as airfoils with varying thickness distribution has been determined and validated through a combination of theoretical methods and computational tools. The definition of a stagnation point is the precise location where the flow velocity is reduced to zero. It is located in the vicinity of the leading edge of an airfoil, but its position moves as a function of the angle of attack and other factors. The stagnation point represents the location of the start of the leading edge expansion on an airfoil surface [1]. This leading edge expansion is the main source of both the suction pressure and the maximum velocity on the upper surface. The leading edge expansion is also strongly related to lift force. Even though the stagnation point location is important in the determination of lift force and flow control applications, there are few known attempts at a simplified approach to its determination on airfoils. A method for the rapid determination of the location of the stagnation point through the application of a generalized empirical equation will be described in the following sections.

Developing the Generalized Empirical Equation

The variable D_s is defined to better describe the precise location of the stagnation point. D_s represents the distance along the airfoil surface from the front of the geometrical leading edge to the stagnation point on an airfoil displayed in Fig. 1.

The generalized empirical equation reflects the relationship between the stagnation point location as a function of the angle of attack, airfoil thickness, degree of camber, and the location of maximum camber. The commercial airfoil design software Designfoil Pro (R5.32) developed by DreeseCode Software [2] and the conformal mapping method [3] are used initially to predict the stagnation point for NACA 4 digit airfoils. The development will begin with the equation for the stagnation point location on a flat plate developed by conformal mapping.

Stagnation Point on a Flat Plate

According to Katz and Plotkin [4] the stagnation point on a flat plate can be expressed with conformal mapping and the Joukowski transformation.

Figure 2a represents the z plane (x, y coordinates) and Fig. 2b represents the w plane (ξ, η coordinates). The classical result from conformal mapping provides $\mu = \beta = 0$ for a flat plate. Thus, the stagnation point location for a flat plate as a function of the angle of attack is

$$x = -\frac{c}{2}\cos(2\alpha) \tag{1}$$

where $c =$ chord, and $\alpha =$ angle of attack.

When plotted (Fig. 3), the trend in D_s is nearly parabolic with an increase in the angle of attack.

Stagnation Point on Symmetrical Airfoils

The conformal mapping method was used to determine the correlation between D_s and both angle of attack and airfoil thickness for symmetrical airfoils. Designfoil was also used to validate the results.

Example of Conformal Mapping Using the NACA 0020

According to Ladson [3], ordinates for the NACA 0020 are described by an equation of the form:

$$\frac{y}{c} = a_0\left(\frac{x}{c}\right)^{0.5} + a_1\left(\frac{x}{c}\right) + a_2\left(\frac{x}{c}\right)^2 + a_3\left(\frac{x}{c}\right)^3 + a_4\left(\frac{x}{c}\right)^4 \tag{2}$$

where $a_0 = -0.2969$, $a_1 = 0.1260$, $a_2 = 0.3516$, $a_3 = -0.2843$, and $a_4 = 0.1015$ for the lower surface.

According to the concept of conformal mapping, all x coordinates on the flat plate can be represented as projections onto the surface of the NACA 0020 at the corresponding x locations from Eq. (2) as shown in Fig. 4.

Using the equation for D_s for the flat plate obtained in the previous section, the chord normalized x coordinate along with the angle of attack can be used to calculate the stagnation point location on the symmetrical airfoil. Four prestall angles of attack (4, 6, 8, 10 deg) are selected.

Because Eq. (2) is continuous on the x axis, an arc length formula can be used to estimate the distance between the leading edge and the stagnation point on the surface (D_s) [5]:

$$D_s = \int_0^{d_x} \sqrt{1 + [f'(x)]^2}\,\mathrm{d}x$$

where d_x represents the distance along the surface from the leading edge to the stagnation point on the x axis. Then D_s becomes

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$$D_s = \int_0^{d_x} \sqrt{1 + [(-0.297(x)^{0.5} + 0.126(x) + 0.352(x)^2 - 0.284(x)^3 + 0.102(x)^4)']^2}\,\mathrm{d}x \tag{3}$$

where the units of D_s are the percentage of chord length.

Hence, D_s as a function of the angle of attack is obtained, when substituting the x coordinates into Eq. (3). The x and y coordinates of the stagnation point as a function of the angle of attack were extracted using Designfoil to validate the method. Substituting the airfoil x coordinates into Eq. (3), the D_s are obtained as displayed in Fig. 5. Comparing the two methods at four angles of attack shows good agreement (93.4% average).

Generalized Equation

In the same manner, airfoil thickness effects were isolated in the determination of a generalized equation using four airfoils, the NACA 0001, 0012, 0016, and 0020. Figure 5 represents the value of D_s as a function of varying angles of attack. Plotting these results across the thickness-to-chord ratios studied reveals clear trends in D_s for a given angle of attack.

Figure 6 shows two examples of D_s as a function of varying airfoil thicknesses for two different angles of attack. The resulting slopes and the intercepts are used to obtain generalized equations as shown in Fig. 7.

The resulting equations for the curve fits are $y = [(-0.9\alpha^2 + 20\alpha - 2)t] \cdot 10^{-5}$ for the slope and $y = (30\alpha^2 - 30\alpha + 6) \cdot 10^{-5}$ for the Y intercept. Combining the two equations, the generalized equation for D_s for symmetrical airfoils is

$$D_S = [(-0.9\alpha^2 + 20\alpha - 2)t + 30\alpha^2 - 30\alpha + 6] \cdot 10^{-5} \tag{4}$$

where α represents the angle of attack and t represents the airfoil thickness in percent.

Stagnation Point Location on Cambered Airfoils

There are two additional factors necessary to plot the profile of cambered airfoils when compared to the symmetric airfoils already presented: one is the camber ratio and the other is the location of the maximum camber. The camber line equation becomes

$$\frac{y}{c} = \frac{p}{m^2}\left[\left(2m\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right] \tag{5}$$

forward of the maximum ordinate

$$\frac{y}{c} = \frac{p}{(1-m)^2}\left[(1-2m) + \left(2m\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right] \tag{6}$$

aft of the maximum ordinate

Example for the Cambered Wing Case, Conformal Mapping with the NACA 4412

Coefficients for a thickness-to-chord ratio of 12% are $a_0 = 0.17574$, $a_1 = -0.0756$, $a_2 = -0.21096$, $a_3 = 0.17058$, and

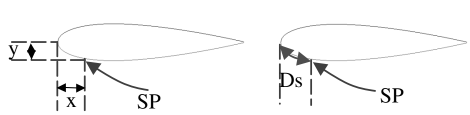
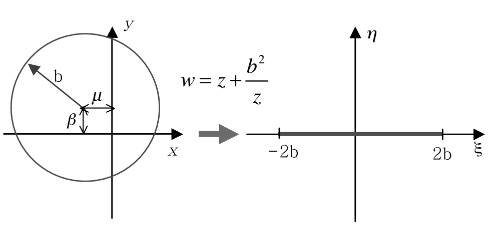


Fig. 1 Graphical explanation of the definition of D_s.



a) z plane b) w plane

Fig. 2 Joukowski transformation: mapping of the circle to a flat plate.

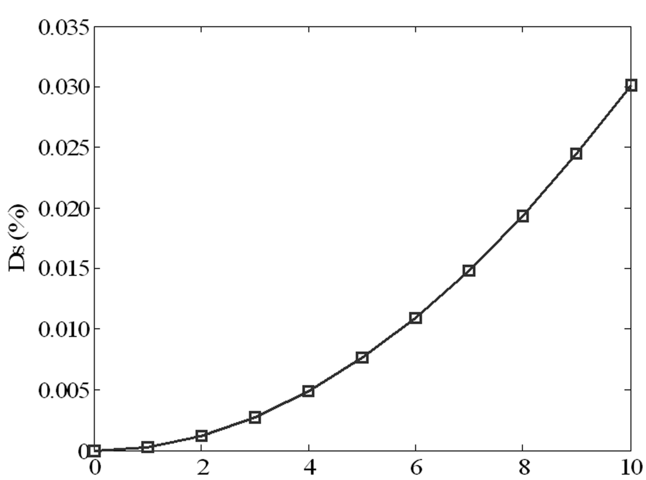


Fig. 3 Trend in D_s for a flat plate from classical aerodynamic theory.

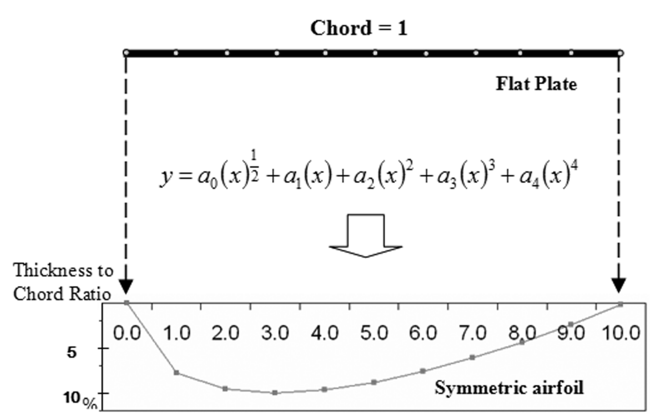


Fig. 4 Conformal mapping.

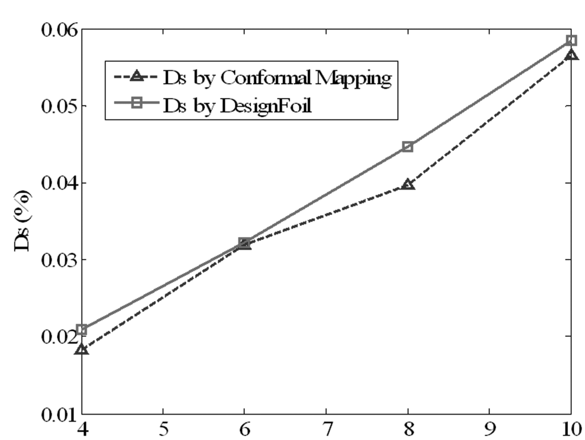


Fig. 5 D_s from Designfoil and conformal mapping for a NACA 0020.

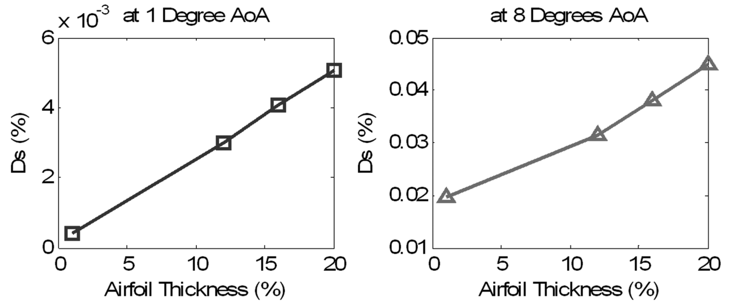


Fig. 6 Examples of the resulting curves for D_s as a function of thickness-to-chord ratio at two representative angles of attack.

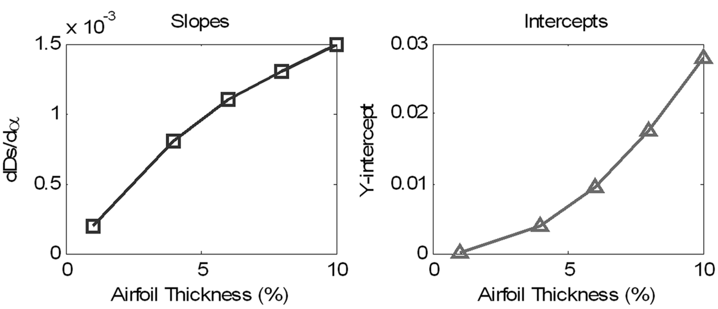


Fig. 7 Variation of D_s as a function of thickness-to-chord ratio (slope) and y intercept as a function of thickness-to-chord ratio.

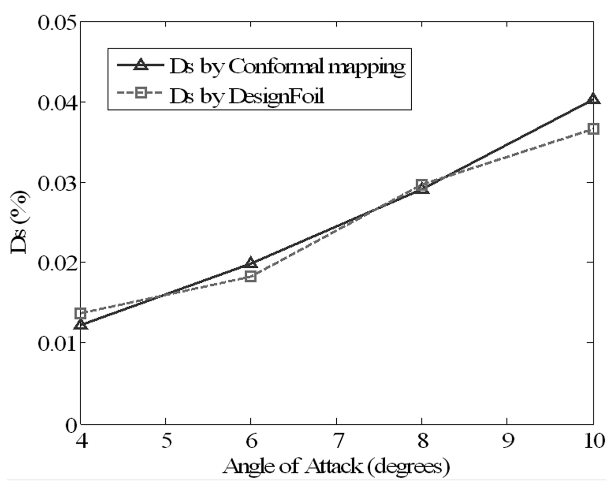


Fig. 8 D_s from Designfoil and conformal mapping with NACA 4412.

$a_4 = -0.0609$. Substituting these coefficients into Eqs. (5) and (6), the equation for the NACA 4412 surface is

$$y = \left[\frac{0.04}{0.4^2}[(2 \cdot 0.4x) - (x)^2]\right] - \left[0.17575(x)^{\frac{1}{2}} - 0.0756\left(\frac{x}{c}\right) - 0.21096(x)^2 + 0.17058(x)^3 - 0.0609(x)^4\right] \tag{7}$$

As was done previously, four angles of attack are selected to obtain D_s (Fig. 8). Comparing the two methods for four angles of attack again shows good agreement (92.6% average).

Generalized Equation

The effects of the degree of camber and the location of maximum camber should be considered in obtaining a generalized equation for the stagnation point location. When an analysis similar to that performed in previous sections is performed (Fig. 9) D_s has a weak dependence on the degree of camber (the maximum difference is less than $\pm 4\%$). As can be seen in Fig. 10, D_s has a weak dependence on the location of the maximum camber as well (the difference is less than $\pm 1.5\%$ on average). It was therefore concluded that it was not necessary to include the degree of camber and the location of

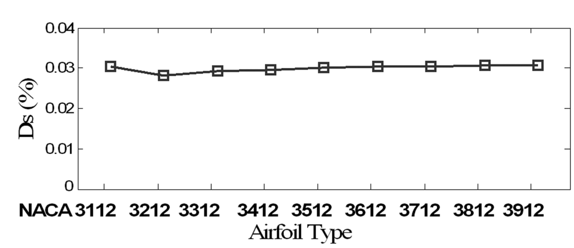


Fig. 10 Representative curve demonstrating the effect of the maximum camber location on the stagnation point location at an angle of attack of 8 deg.

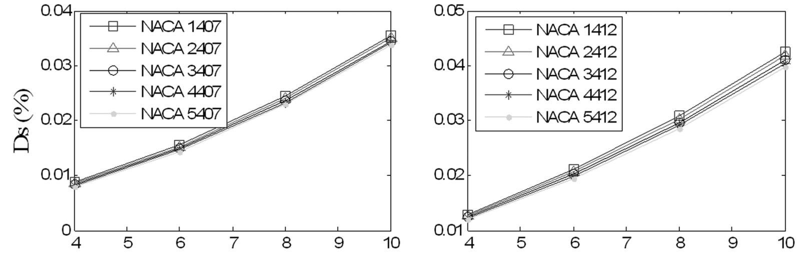


Fig. 9 Effect of the degree of camber on the stagnation point location.

maximum camber in the generalized equation for cambered airfoils. The angle of attack and thickness-to-chord ratio were found to be the most relevant factors in the calculating D_s. Thus the resulting generalized equation for the stagnation point location on NACA airfoils is

$$D_S = [(-0.9\alpha^2 + 20\alpha - 2)t + 30\alpha^2 - 30\alpha + 6] \cdot 10^{-5}$$

where $\alpha =$ angle of attack, and $t =$ airfoil thickness.

To validate this equation, D_s was calculated using Designfoil and the agreement of the empirical equation is within $\pm 5\%$.

Conclusions

A generalized method for the prediction of the stagnation point location as a function of the angle of attack on symmetrical and cambered wing profiles as well as varying thickness distribution has been determined and validated through a combination of theoretical methods and computational tools.

The generalized empirical equation for both the symmetrical and cambered airfoils is found to be

$$D_S = [(-0.9\alpha^2 + 20\alpha - 2)t + 30\alpha^2 - 30\alpha + 6] \cdot 10^{-5}$$

where $\alpha =$ angle of attack, and $t =$ airfoil thickness in percent.

This equation is a function of the thickness-to-chord ratio and the angle of attack alone because the stagnation point location showed a weak dependence on both the degree of camber and the location of maximum camber.

In conclusion, a generalized empirical equation for the location of the stagnation point on airfoils has been obtained through the combined use of commercial airfoil design software and complex conformal mapping. The resulting empirical approach allows for a rapid determination of the airfoil stagnation point location without the need for a complete airfoil flowfield analysis or flowfield analysis tools.

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